Practical Commitment of Combined Cycle Plants using Dynamic Programming

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***Abstract*—Due to the existence and building of an important number of combined cycle plants throughout electric power systems around the world, there exists the growing need to have a more accurate model to represent these type of power plants when solving the unit commitment problem. A commonly used optimization technique to solve the unit commitment problem is dual programming. This work focuses on solving the subproblem of scheduling a combined cycle plant using dynamic programming under a dual optimization scheme. The model used to represent the combined cycle plants is based on configurations; this new model takes into account such constraints as the feasible transitions between configurations, and the minimum and maximum time that a combined cycle plant must remain on a certain configuration. This model accurately represents the real-life characteristics of combined cycle plants like different start-up sequences and different stopping conditions. One novelty of this model is that the representation of each one of the states and configurations is done with a single integer state index that consecutively sums the time that the combined cycle plant must remain on each state or configuration. The use of this integer state index simplifies the state-space diagrams and reduces the number of integer/binary variables in the model. Another novelty is the modeling of Hybrid Combined Cycle Plants; these are the ones that use an auxiliary boiler in order to increase the production of steam.**

***Index Terms*—Combined cycle plant, configurations, dual programming, dynamic programming, transitions, unit commitment.**

I. Introduction

Due to the recent changes in the electricity industry such as deregulation, the opening of the electricity market to private investors, and an increased concern for the environment, an important number of combined cycle plants (CCPs) have been built all over the world. This, in turn, has brought new challenges to the unit commitment (UC) problem. The challenges mainly come from the need to model the CCPs in a more accurate way and then incorporate this model to the UC problem.

The short-term UC problem is basically to determine, for each hour of the next day and for up to seven days, the optimum operating point of the available hydro and thermal units in order to satisfy the forecasted level of demand with the minimum operating cost and, while doing so, meeting all the physical and operational constraints of the power system. Some of the more common constraints incorporated to the UC problem are load balance, spinning reserve, scheduled reserve, off-line reserve, must-run units, and fuel consumption among others. Constraints that are particular to thermal units include minimum and maximum up and down time limits, start-up costs, and minimum and maximum generation limits. When CCPs are incorporated to the UC problem, the following constraints must be considered: different configurations of the CCPs, feasible transitions between configurations, and transitions costs between configurations. For the regulated electricity industry, the objective when solving the UC problem is to minimize the operating costs while satisfying the demand whereas for the deregulated electricity industry, the goal of market participants is to maximize their benefits rather than satisfying the demand at a minimum cost. When network constraints are incorporated to the UC problem, and using an AC formulation, the UC problem is known as security constrained unit commitment (SCUC) [1]–[4].

This paper is organized as follows. Section II explains the operational and technical constraints of Combined Cycle Plants. The different models used to represent the Combined Cycle Plants are discussed in Section III. Section IV presents the different existent approaches used to solve the Unit Commitment problem when incorporating Combined Cycle Plants. The new model based on configurations using Dynamic Programming for Combined Cycle Plants under a Dual Programming scheme is detailed in Section V. Section VI presents a numerical example to illustrate the use of the new model for Combined Cycle Plants based on configurations. Some concluding remarks are given in Section VII.

II. Combined Cycle Plants

Combined cycle plants are generating units that have flexible operating conditions. Other generating units with flexible operating conditions are: i) fuel switching units, ii) fuel blending units, iii) constant/variable pressure units, iv) over-fire units, and v) dual boiler units [1], [5]. CCPs are made up of one or more combustion turbine (CT) generators, each one of them with its own heat recovery steam generator (HRSG), and one conventional steam turbine (ST) generator common to all the CTs. Some CCPs may have an auxiliary boiler in order to generate more steam to aid the HRSG to drive, or even drive independently, the ST. This type of CCPs are called Hybrid Combined Cycle Plants (HCCPs); this is shown in Figure 1. The most basic CCP is formed by one CT generator, one

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HRSG, and one ST generator, and its operation is as follows: In the first stage a mixture of air and fuel is burned in the combustor of the CT. The released energy by the combustion is used to move the CT which in turn moves a generator to generate electricity. In the second stage the hot gases of the CT, that otherwise would be wasted to the atmosphere, are used by the HRSG to generate steam and that steam is used to move the ST that in turn moves a generator to produce electricity. This configuration of a CCP can reach an efficiency of up to 60%; this is an improvement of 20% - 30% with respect to conventional combustion turbines [2]. CCP have high thermal efficiency, that is, a lesser fuel consumption to generate the same energy. Less fuel consumption implies lower operating costs and and less emission of pollutants per unit of energy generated making them not only an economical option but also an environmentally friendly solution [5], [6].

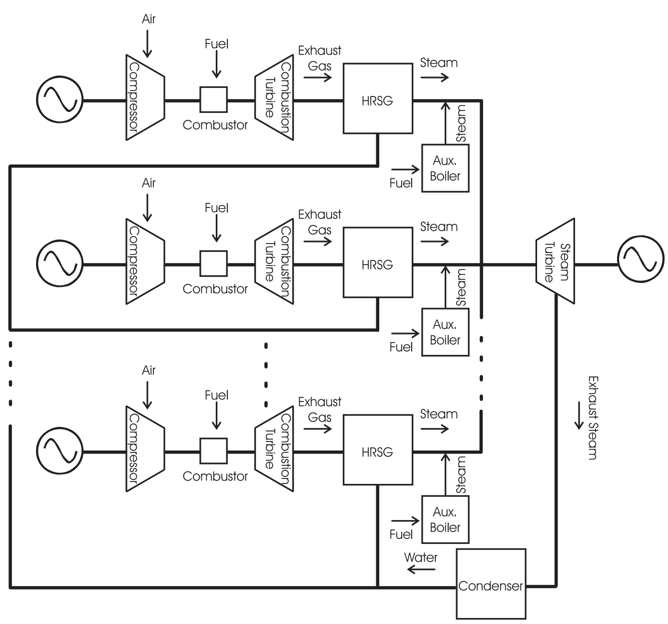

Fig. 1. Schematics of a CCP with Auxiliary Boiler

CCPs have different *configurations* and each configuration may have several *states*. The generation capacity of a CCP changes depending on which configuration the CCP is on, and the transition costs from one configuration to another are by no means negligible. CCP configurations are dependent on the different combinations that can be formed with the CT generators, HRSGs, and the ST generator; for instance, CT generators can be operated with or without their associated HRSGs whereas the ST generator cannot be operated without at least one HRSG available. The number of states for a given configuration is the number of periods, *i.e.* hours, that the CCP must remain on that configuration. If it takes for a given CCP three periods to complete the cold start-up sequence, then the start-up configuration has three states. The transitions between configurations must follow operational and time constraints; these are imposed by the number of periods that a CCP must remain on a given configuration and also by the rules that dictate to which configuration it is feasible to transition to [1], [2], [5].

III. Modeling of Combined Cycle Plants

There are at least three different ways to model CCPs. These are: i) aggregated model, ii) model based on configurations, and iii) model based on physical components. A brief descriptions of each one of them is presented next.

The *aggregated model* ignores all the components of a CCP and considers it as a single equivalent unit; hence, the scheduling of the CCPs is done as if they were conventional thermal units. This is a very simplistic model since the commitment of the equivalent unit results in an on-off state therefore ignoring all the different configurations and technical constraints of the CCP. At the end, the configuration of the CCP in real time is left to the plant operators. This model is currently in use by ISO NE, NYISO, MISO, and PJM [7]. The model currently used by the Electric Research Institute of México (IIE, acronym in spanish) is also an aggregated model. CCPs are modeled as equivalent thermal units whose power output is the sum of all the CT generators affected by a contribution factor to the ST generator. Even though the aggregated model is a "working" representation of a CCP, a more accurate model is needed to account for the different configurations and for the start-up sequences; this is the model based on configurations.

The *model based on configurations* considers the three major components of a CCP, namely CT generators, HRSGs, and ST generator. Considering these three major components of the CCP allows the representation of the minimum and maximum on and off time, transition time, start-up sequences, load ramp rates, minimum and maximum power output limits by configuration, individual heat energy requirement curves per configuration, start-up costs, and transition costs between configurations. All the possible transitions between configurations are represented by space-state diagrams. Another advantage of this model is that the availability of each one of these components can be taken into account.

IV. Solution Techniques

There are several solution techniques to solve the UC and SCUC problems when CCPs are included in the formulation. Some of these techniques are Mixed Integer Programming (MIP) [7]; evolutionary algorithms like Genetic Algorithms (GA), Evolutionary Programming (EP), and Particle Swarm Optimization (PS) [8]; and Dual Programming combined with Dynamic Programming. A variation of the latter one, called Dynamic Programming with Lagrangian Reduction of Search-Range, is proposed by [9]. The Lagrangian Relaxation (LR) technique decomposes the UC problem into a set of sub-problems in which the coupling system constraints are relaxed and included in the objective function by means of their lagrange multipliers. Each sub-problem, one subproblem per individual generating unit, minimizes the generation costs, start-up costs, and the terms including the lagrange multipliers subject to minimum and maximum on and off time constraints, minimum and maximum generating level constraints, and ramp rate constraints. These sub-problems are solved individually using DP; the commitment of each one of the generating units is done for all the periods considered in the planning horizon [1]–[3]. The techniques used to solve the SCUC problem are Benders' Decomposition and Lagrangian Relaxation with Dynamic Programming. In [5], the SCUC problem is divided into two problems using Benders' Decomposition. The master problem solves the unit commitment problem while the slave problem minimizes the network violations. The master problem is solved with LR and DP using transition diagrams for the CCPs.

This work focuses on the solution, using DP, of the sub-problem of committing the CCPs based on a configuration model under a dual programming scheme, in other words, the goal is to find the configuration of the CCPs for each one of the periods in the planning horizon.

V. Combined Cycle Plants based on a Configuration Model under a Dual Programming Scheme

When scheduling CCPs using DP, all the configurations that the CCP has, along with their states, minimum and maximum number of periods for each configuration, transition costs between configurations and states, and the set of feasible transitions between configurations and states must be defined. Also, for each configuration, including all its states, its own minimum and maximum generation levels must be defined as well.

The new proposed model solves the problem of scheduling a CCP using forward Dynamic Programming (DP) where the lagrange multipliers for the coupling constraints can be obtained for each period of the planning horizon using the well-know lagrangian relaxation (LR) technique [10]. A number of lagrange multipliers from the coupling constraints can be added to the objective function of the individual CCP commitment problem. The most representative coupling constraint in any unit commitment problem is the total power balance constraint [4].

The cost of being in configuration/state I in period K, used in the formulation of the DP problem, is obtained by solving the optimization problem shown in Equations 1 and 2.

$$\min_{P_{u,K}} \quad \sum_{u\in U_I} \Big[a_u P_{u,K}^2 + b_u P_{u,K} + c_u - \lambda_K P_{u,K} \left(1+\sigma_I\right) \Big] \qquad (1)$$

$$\text{s.t.} \quad \underline{P}_{u,K} \leq P_{u,K} \leq \overline{P}_{u,K}, \ \forall u \in U_I, \qquad (2)$$

where a_u, b_u, and c_u are the coefficients of the quadratic cost function derived from the input-output curves of the CT generators, u is the unit, U_I is the set of units that belong to configuration I, λ_K is the lagrange multiplier that is associated with the power balance constraint, $P_{u,K}$ is the power output of unit u in period K, and σ_I is the contribution factor. The contribution factor σ_I is a dimensionless factor that when multiplied by the generation output of the CT generator gives as a result the contributed generation output of the ST generator due to the exhaust gases of the CT generator. The total power output of the CCP in configuration I is the sum of all the power outputs of the CT generators pertaining to configuration I plus the generation of the ST generator due to the contribution of the CTs. This is shown in Equation 3, where the assumption that all the CTs are equal has been made, and P_{CCP_I} represents the total power output of the CCP while in configuration I.

$$P_{CCP_I} = \sum_{u\in U_I} (1+\sigma_I) P_u, \qquad (3)$$

From Equation 3, one can see that the contribution factor σ_I is defined as:

$$\sigma_I = \frac{P_{CCP_I}}{\sum_{u\in U_I} P_u} - 1. \qquad (4)$$

One contribution factor is obtained for each configuration of the CCP.

The generation limits for the CCP are as shown in Equation 5.

$$\sum_{u\in U_I} (1+\sigma_I)\underline{P}_u \leq P_{CCP_I} \leq \sum_{u\in U_I} (1+\sigma_I)\overline{P}_u. \qquad (5)$$

In the case of the HCCPs, and depending on the configuration and output of the exhaust gasses of the CTs, the HRSG may not be able to generate enough steam to drive the ST generator and meet its minimum generation limit. When this happens, the auxiliary boiler must be used (supplementary heat, SH) in order to meet the minimum generation limit of the ST generator. The use of the auxiliary boiler comes at the cost of reducing the efficiency of the HCCP. The cost of being in configuration/state I in period K for a HCCP is shown in Equation 6.

$$\min_{P_{u,K},P_{SH,K}} \quad \sum_{u\in U_I} \Big[a_u P_{u,K}^2 + b_u P_{u,K} + c_u - \lambda_K \left(1+\sigma_I\right) P_{u,K} \Big] + a_{SH} P_{SH,K}^2 + b_{SH} P_{SH,K} - \lambda_K P_{SH,K}, \qquad (6)$$

where $P_{SH,K}$ is the component of the power output of the ST generator due to the use of supplementary heat, and a_{SH} and b_{SH} are the coefficients of the quadratic cost function derived from the input-output curve of the ST generator when using supplementary heat. The discussion on how the input-output curve is obtained for a HCCP is out of the scope of this paper.

The power generation limits for the CT generators remain as in Equation 2. When the exhaust gasses of the CTs are not enough to satisfy the minimum generation requirement of the ST generator, the power limits of how much power is to be generated using SH are as shown in Equation 7. When the exhaust gasses of the CTs are enough to satisfy the minimum generation requirement of the ST generator, the power limits of how much power is to be generated using SH are as shown in Equation 8.

$$\underline{P}_{ST,K} - P_{R,K} \leq P_{SH,K} \leq \overline{P}_{ST,K} - P_{R,K}, \quad (7)$$

$$0 \leq P_{SH,K} \leq \overline{P}_{ST,K} - P_{R,K}, \quad (8)$$

where $\underline{P}_{ST,K}$ and $\overline{P}_{ST,K}$ are the minimum and maximum generation limits of the ST generator, $P_{SH,K}$ is the power output of the ST generator due to the SH, and $P_{R,K} = \sum_{u \in U_I} \sigma_I P_{u,K}$ is the power output of the ST generator due to exhaust gasses of CTs (residual).

The total power output of a HCCP while in configuration I is shown in Equation 9.

$$P_{HCCP_I} = \sum_{u \in U_I} (1 + \sigma_I) P_u + P_{SH}. \quad (9)$$

One of the novelties of this model is that the configurations/states are represented by only one integer state index that, depending on its value, it is able to identify each and every one of the different configurations/states. This is accomplished by assigning a value of 1 to any of the configurations at the start and then, consecutively, adding the number of periods that it takes to complete each and every one of the configurations/states for the CCP. The use of an integer state index has the following advantages: When representing the minimum and maximum time for each configuration, the number of variables and the number of states in the state transition diagram do not increase. Hence, the state-space diagrams using this formulation are much simpler. Another advantage is that this formulation does not need integer variables in order to keep track of the number of hours that a CCP has been on a given configuration nor does it need binary start-up/sut-down variables. The use of the integer state index is shown in detail in the following numerical example.

VI. Numerical Example

Consider a HCCP (supplementary heat) that has three CT generators and one ST generator. The planning horizon is five hourly periods. It is assumed that the contribution factor, $\sigma_I = 0.409639$, and the cost functions, as shown in Table I, for the CT generators and the supplementary heat remain constant over the planing horizon regardless of the configuration of the HCCP.

TABLE I
Unit cost functions and generation limits.

Unit	$a^{\dagger}$	$b^{\ddagger}$	c^{*}	$\underline{P}^{\diamond}_{K,I}$	$\overline{P}^{\diamond}_{K,I}$
Combustion Turbine 1	2.9707	118.747	20967.334	65	83
Combustion Turbine 2	2.9766	121.814	20997.162	65	83
Combustion Turbine 3	2.9826	118.950	21049.381	65	83
Steam Turbine	×	×	×	150	300
Supplementary Heat	0.2260	417.866	×	×	×

† in $/MW^{2}h, ‡ in $/MWh, * in $/h, y $^{\diamond}$ in MW.

TABLE II
Configurations and States.

Configuration	State	No. CTs	ST	SH
CSUS	1	0	×	×
	2	1	✓	×
	3	2	✓	×
	4	3	✓	×
WSUS	1	1	✓	×
	2	2	✓	×
	3	3	✓	×
HSUS	1	3	✓	×
CS	$t^{\min}_{\text{CS}}$	0	×	×
WS	$t^{\min}_{\text{WS}} \leq t_{\text{WS}} \leq t^{\max}_{\text{WS}}$	0	×	×
HS	$t^{\min}_{\text{HS}} \leq t_{\text{HS}} \leq t^{\max}_{\text{HS}}$	0	×	×
1 CT + ST	$t^{\min}_{\text{1CTST}}$	1	✓	×
1 CT + ST + SH	$t^{\min}_{\text{1CTSTSH}}$	1	✓	✓
2 CT + ST	$t^{\min}_{\text{2CTST}}$	2	✓	×
2 CT + ST + SH	$t^{\min}_{\text{2CTSTSH}}$	2	✓	✓
3 CT + ST	$t^{\min}_{\text{3CTST}}$	3	✓	×
3 CT + ST + SH	$t^{\min}_{\text{3CTSTSH}}$	3	✓	✓

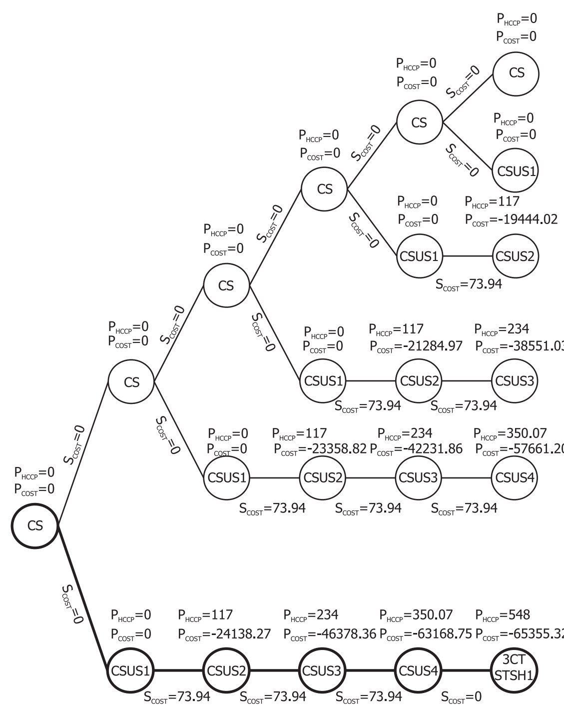

Fig. 2. Dynamic Programming Solution.

Three start-up sequences are considered: Cold Start-up Sequence (CSUS), Warm Start-up Sequence (WSUS), and Hot Start-up Sequence (HSUS); the number of hourly periods that each one of the starp-up sequences take until their completion is also considered. The minimum and maximum number of hourly periods that a HCCP must remain in a given configuration are considered as well. In Table II the configurations and states for this numerical example are given, where $t^{\min}_{\text{CS}} = 4$ is the minimum time that the HCCP must remain in the Cold Stop configuration; $t^{\min}_{\text{WS}} = 4$ and $t^{\max}_{\text{WS}} = 5$ are the minimum and maximum time that the HCCP must remain in the Warm Stop configuration, respectively; $t^{\min}_{\text{HS}} = 4$ and $t^{\max}_{\text{HS}} = 5$ are the minimum and maximum time that the HCCP must remain

TABLE III
INTEGER INDEX χ_K, A.

χ_K	Configuration/State
1	HS
2	
...	
$t_{\text{HS}}^{\text{min}}$	
$t_{\text{HS}}^{\text{min}} + 1$	
...	
$t_{\text{HS}}^{\text{max}}$	
$t_{\text{Hs}}^{\text{max}} + 1$	WS
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{min}}$	
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}}$	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + 1$	CS
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}}$	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + 1$	CSUS
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}}$	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + 1$	WSUS
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}}$	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + 1$	HSUS
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}}$	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + 1$	1 CT+ST
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + t_{\text{1CTST}}^{\text{min}}$	

TABLE IV
INTEGER INDEX χ_K, B.

χ_K	Configuration/State
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + t_{\text{1CTST}}^{\text{min}} + 1$	1 CT+ST+SH
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + t_{\text{1CTST}}^{\text{min}} + t_{\text{1CTSTSH}}^{\text{min}}$	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + t_{\text{1CTST}}^{\text{min}} + t_{\text{1CTSTSH}}^{\text{min}} + 1$	2 CT+ST
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + t_{\text{1CTST}}^{\text{min}} + t_{\text{1CTSTSH}}^{\text{min}} + t_{\text{2CTST}}^{\text{min}}$	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + t_{\text{1CTST}}^{\text{min}} + t_{\text{1CTSTSH}}^{\text{min}} + t_{\text{2CTST}}^{\text{min}} + 1$	2 CT+ST+SH
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + t_{\text{1CTST}}^{\text{min}} + t_{\text{1CTSTSH}}^{\text{min}} + t_{\text{2CTST}}^{\text{min}} + t_{\text{2CTSTSH}}^{\text{min}}$	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + t_{\text{1CTST}}^{\text{min}} + t_{\text{1CTSTSH}}^{\text{min}} + t_{\text{2CTST}}^{\text{min}} + t_{\text{2CTSTSH}}^{\text{min}} + 1$	3 CT+ST
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + t_{\text{1CTST}}^{\text{min}} + t_{\text{1CTSTSH}}^{\text{min}} + t_{\text{2CTST}}^{\text{min}} + t_{\text{2CTSTSH}}^{\text{min}} + t_{\text{3CTST}}^{\text{min}}$	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + t_{\text{1CTST}}^{\text{min}} + t_{\text{1CTSTSH}}^{\text{min}} + t_{\text{2CTST}}^{\text{min}} + t_{\text{2CTSTSH}}^{\text{min}} + t_{\text{3CTST}}^{\text{min}} + 1$	3 CT+ST+SH
...	
$t_{\text{HS}}^{\text{max}} + t_{\text{WS}}^{\text{max}} + t_{\text{CS}}^{\text{min}} + t_{\text{CSUS}} + t_{\text{WSUS}} + t_{\text{HSUS}} + t_{\text{1CTST}}^{\text{min}} + t_{\text{1CTSTSH}}^{\text{min}} + t_{\text{2CTST}}^{\text{min}} + t_{\text{2CTSTSH}}^{\text{min}} + t_{\text{3CTST}}^{\text{min}} + t_{\text{3CTSTSH}}^{\text{min}}$	

the the Hot Stop configuration, respectively; $t_{\text{1CTST}}^{\text{min}} = 4$ is the minimum time that the HCCP must remain in the 1 CT generator and ST generator configuration; $t_{\text{1CTSTSH}}^{\text{min}} = 4$ is the minimum time that the HCCP must remain in the 1 CT generator, ST generator, and SH; $t_{\text{2CTST}}^{\text{min}} = 4$ is the minimum time that the HCCP must remain in the 2 CT generator and ST generator configuration; $t_{\text{2CTSTSH}}^{\text{min}} = 4$ is the minimum time that the HCCP must remain in the 2 CT generator, ST generator, and SH; $t_{\text{3CTST}}^{\text{min}} = 4$ is the minimum time that the HCCP must remain in the 3 CT generator and ST generator configuration; and $t_{\text{3CTSTSH}}^{\text{min}} = 4$ is the minimum time that the HCCP must remain in the 3 CT generator, ST generator, and SH.

The integer state index χ_K, depending on its value, represents each and every single one of the configurations/states of the HCCP. This integer state index adds up consecutively the number of hourly periods for each configuration/state. The value of χ_K for all the configurations/states, after defining $\chi_K = 1$ as the first hourly period of the HS configuration, is shown in Tables III and IV, where t_{CSUS} is the number of hourly periods that it takes for the Cold Start-up Sequence to be completed, t_{WSUS} is the number of hourly periods that it takes for the Warm Start-up Sequence to be completed, and t_{HSUS} is the number of hourly periods that it takes for the Hot Start-up Sequence to be completed.

The feasible transitions between configurations and states are shown in Table V.

Assume that there is a fixed transition cost of \$73.94 every time that a CT is started. The value for λ_K for the five hours of the planning horizon are 706.83\$/MWh, 644.61\$/MWh, 637.95\$/MWh, 620.23\$/MWh, and 604.50\$/MWh, respectively.

The generation, after solving, per component of the HCCP is shown in Table VI. Figure 2 shows the details of the dynamic programming search path. The final commitment of the HCCP, the optimal path, is CS $\rightarrow$ CSUS $\rightarrow$ CSUS $\rightarrow$ CSUS $\rightarrow$ CSUS $\rightarrow$ 3 CT+ST+SH. It can be seen that the transition from the CSUS, when completed, to the 3 CT+ST+SH configuration is imposed since the residual heat from the three CTs is not enough to meet the minimum generation limit of the ST generator, hence it is forced to use SH to meet the minimum generation limit of the ST generator.

The current model was programmed in Fortran and tested embedded into the current planning tool used by the Mexican Electricity Company (Comisión Federal de Electricidad) using

TABLE V
FEASIBLE TRANSITIONS.

<table>
<tr><th>χ_K</th><th>χ_{K-1}</th></tr>
<tr><td rowspan="6">1st interval HS</td><td>1 CT+ST; min. time completed</td></tr>
<tr><td>1 CT+ST+SH; min. time completed</td></tr>
<tr><td>2 CT+ST; min. time completed</td></tr>
<tr><td>2 CT+ST+SH; min. time completed</td></tr>
<tr><td>3 CT+ST; min. time completed</td></tr>
<tr><td>3 CT+ST+SH; min. time completed</td></tr>
<tr><td>HS</td><td>HS</td></tr>
<tr><td>1st interval WS</td><td>HS; max. time completed</td></tr>
<tr><td>WS</td><td>WS</td></tr>
<tr><td>1st interval CS</td><td>WS; max. time completed</td></tr>
<tr><td>CS</td><td>CS</td></tr>
<tr><td>CS</td><td rowspan="2">CS; min. time completed</td></tr>
<tr><td>1st interval CSUS</td></tr>
<tr><td>CSUS</td><td>CSUS</td></tr>
<tr><td>1st interval WSUS</td><td>WS; min. time completed</td></tr>
<tr><td>WSUS</td><td>WSUS</td></tr>
<tr><td>1st interval HSUS</td><td>HS; min. time completed</td></tr>
<tr><td>HSUS</td><td>HSUS</td></tr>
<tr><td rowspan="5">1st interval 1 CT+ST</td><td>1 CT+ST+SH; min. time completed</td></tr>
<tr><td>2 CT+ST; min. time completed</td></tr>
<tr><td>2 CT+ST+SH; min. time completed</td></tr>
<tr><td>3 CT+ST; min. time completed</td></tr>
<tr><td>3 CT+ST+SH min. time completed</td></tr>
<tr><td rowspan="2">1 CT+ST</td><td>1 CT+ST</td></tr>
<tr><td rowspan="2">1 CT+ST min. time completed</td></tr>
<tr><td rowspan="3">1st interval 1 CT+ST+SH</td></tr>
<tr><td>2 CT+ST+SH; min. time completed</td></tr>
<tr><td>3 CT+ST+SH; min. time completed</td></tr>
<tr><td rowspan="2">1 CT+ST+SH</td><td>1 CT+ST+SH</td></tr>
<tr><td>1 CT+ST+SH min. time completed</td></tr>
<tr><td rowspan="5">1st interval 2 CT+ST</td><td>1 CT+ST min. time completed</td></tr>
<tr><td>1 CT+ST+SH min. time completed</td></tr>
<tr><td>2 CT+ST+SH min. time completed</td></tr>
<tr><td>3 CT+ST min. time completed</td></tr>
<tr><td>3 CT+ST+SH min. time completed</td></tr>
<tr><td rowspan="2">2 CT+ST</td><td>2 CT+ST</td></tr>
<tr><td rowspan="2">2 CT+ST min. time completed</td></tr>
<tr><td rowspan="2">1st interval 2 CT+ST+SH</td></tr>
<tr><td>3 CT+ST+SH min. time completed</td></tr>
<tr><td rowspan="2">2 CT+ST+SH</td><td>2 CT+ST+SH</td></tr>
<tr><td>2 CT+ST+SH min. time completed</td></tr>
<tr><td rowspan="5">1st interval 3 CT+ST</td><td>CSUS completed</td></tr>
<tr><td>WSUS completed</td></tr>
<tr><td>HSUS completed</td></tr>
<tr><td>2 CT+ST+SH min. time completed</td></tr>
<tr><td>3 CT+ST+SH min. time completed</td></tr>
<tr><td rowspan="2">3 CT+ST</td><td>3 CT+ST</td></tr>
<tr><td rowspan="2">3 CT+ST min. time completed</td></tr>
<tr><td>1st interval 3 CT+ST+SH</td></tr>
<tr><td rowspan="2">3 CT+ST+SH</td><td>3 CT+ST+SH</td></tr>
<tr><td>3 CT+ST+SH min. time completed</td></tr>
</table>

representative data. Nine CCP and one HCCP were analyzed simultaneously for a planing horizon of 168 hours (one week).

VII. CONCLUSION

The authors present a new model for Combined Cycle Plants and Hybrid Combined Cycle Plants based on configurations using Dynamic Programming under a Dual Optimization Scheme. This new model is an answer to the needs of power system operators by means of providing a more accurate model that is able to represent to the fullest extent possible the technical constraints of a Combined Cycle Plant. Among these technical constraints are the different start-up sequences and the different configurations/states that a Combined Cycle Plant has. The minimum and maximum number of intervals that a Combined Cycle Plant must remain in a given configuraion/state are also considered. An innovation on the modeling of the different sates/configurations is made by using a single integer state index that is able to represent each an every single one of these states/configurations; this simplifies the state-space diagrams and reduces the number of integer/binary variables in the model. A way of modeling Hybrid Combined Cycle Plants, the ones with a dual boiler to produce supplementary heat, is also proposed.

TABLE VI
GENERATION PER COMPONENT.

Hour	CT 1†	CT 2†	CT 3†	ST†		HCCP †
				SH	Res.	
In. Cond.	0.0	0.0	0.0	0.0	0.0	0.0
1	0.0	0.0	0.0	0.0	0.0	0.0
2	83.0	0.0	0.0	0.0	34.0	117.0
3	83.0	83.0	0.0	0.0	68.0	234.0
4	83.0	82.3	83.0	0.0	101.7	350.0
5	83.0	82.3	82.7	198.4	101.6	548.0

† in MW.

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